Walsh Function Based Synthesis Method of PWM Pattern for Full-Bridge Inverter

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Abstract—This paper applies the Walsh function to simplify the on-off pattern synthesis of a single phase full-bridge PWM inverter with low harmonics. Owing to the digital nature of the Walsh function, the synthesis process becomes straightforward. It is the remarkable feature of the proposed method over a conventional method using Fourier series. Several examples are shown in the paper, which verifies the validity of the proposed synthesis method.

I. INTRODUCTION

The Fourier series expansion is conventionally used to derive optimal PWM pattern with low harmonics [1]. The trigonometric function used in the Fourier series causes a problem. It is that an equation including trigonometric function requires a numerical iteration process to find its solution. The iteration loop shown in Fig. 1 is necessary for searching optimal PWM pattern with low harmonics and it takes long calculation time.

Fig.1 Outline for searching optimal PWM pattern

The Walsh function has only two level signal of ±1 and is suitable for the application to switching process [2]. As it will be described in section III, the equation relating the on-off switching timing with the Walsh spectrum results in a liner algebraic equation which can be solved directly without any iteration process. Hence, the PWM pattern synthesis using the Walsh function becomes straightforward and its computation is simpler and faster than that using trigonometric function.

II. FUNDAMENTALS OF WALSH FUNCTION

A. Definition of Walsh Function

The Walsh function forms an ordered set of rectangular waveforms taking only two amplitude values, +1 and -1, over one normalized frequency period [0, 1] as illustrated in Fig.2. The Walsh function, wal(n,t), has Walsh’s original order, where n is the number of zero axis crossing for one cycle. There are many way to create wal(n,t). In this paper, the Walsh function wal(n,t) is indirectly generated by the recurrence relation of H():

\[ H(0,t) = \begin{cases} 1 & -\frac{1}{2} \leq t \leq \frac{1}{2} \\ 0 & \text{otherwise} \end{cases} \]

\[ H(2n+p,t) = (-1)^{(n/2)+p} \{H(n,2(t+\frac{1}{4}))\} + (-1)^{n+p} \{H(n,2(t-\frac{1}{4}))\} \]  (2)

where, \([n/2]\) is maximum integer that is smaller than \(n/2\), and \((m\%2)\) is the reminder of \(m\) divided by 2. Then the function H() gives the Walsh function wal() as:

\[ \text{wal}(n,t) = H(2n+p,t). \]  (3)

Similar to the trigonometric function, the Walsh function can be classified into odd-function and even-function:

\[ \text{sal}(m,t) = \text{wal}(2m-1,t) \quad \text{[odd-function]}, \]
\[ \text{cal}(m,t) = \text{wal}(2m,t) \quad \text{[even-function]} \]  (4)

The sal() and cal() are corresponding to sine-Walsh and cosine-Walsh, respectively. Unlike the sine and cosine functions in the Fourier series, the sal() and cal() have rectangular waveforms. This kind of binary nature of the Walsh function makes the calculation of the series expansion simpler than that of the Fourier series expansion.
B. Walsh Series Expansion

An appropriately continuous function \(x(t)\) can be expressed by the Walsh series in the similar way as the Fourier series expansion:

\[
x(t) = A_0 \text{wal}(0,t) + \sum_{i=1}^{\infty} \{ A_i \text{cal}(i,t) + B_i \text{sal}(i,t) \},
\]

(5)

where,

\[
\begin{align*}
A_0 &= \int_0^1 x(t) \, dt \\
A_i &= \int_0^1 x(t) \text{cal}(i,t) \, dt \\
B_i &= \int_0^1 x(t) \text{sal}(i,t) \, dt
\end{align*}
\]

(6)

C. Comparison between Fourier and Walsh Expansions of PWM pattern

In order to compare calculation time between Fourier and Walsh expansions, it is assumed that the \(f(t)\) is a PWM switching pattern of which first quarter cycle is shown in Fig. 3(b) and whole one cycle is shown in Fig. 3(a). The Fourier and Walsh series-expansions can be derived as follows.

Walsh series expansion:

\[
f(t) = \sum_{i=1,3,5,...}^{\infty} B_i \text{sal}(i,t)
\]

\[
B_i = \frac{1}{4} \int_0^{\frac{\pi}{4}} f(t) \text{sal}(i,t) \, dt + \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} f(t) \text{sal}(i,t) \, dt
\]

for \(i=1,\)

\[
B_i = \frac{1}{4} \left( (\alpha_2 - \alpha_4) + (\alpha_4 - \alpha_2) \right).
\]

(8)

For the Fourier expansion, the computation of \(C_i\) in (7) requires the calculation of \(\cos()\), which takes relatively long time. On the other hand, for the Walsh expansion, the computation of \(B_i\) in (8) requires only plus/minus operations. This benefit of the Walsh function can make the calculation faster. By the computer simulation result, it was found that the computation time ratio of (8)/(7) was 0.029.

III. PROPOSED SYNTHESIS METHOD OF PWM PATTERN

A. Outline of Proposed Method

Fig. 4 illustrates the outline of the PWM pattern synthesis process proposed in this chapter. The goal of the synthesis is to make the PWM pattern close to the commanded output waveform which is illustrated as a sine-wave in Fig. 4. In the following, it is assumed that the commanded output waveform is a sine-wave to make the explanation simple.

At the first step, the values of \(B_i\), which are the Walsh series coefficients of the commanded waveform, are calculated. This will be described in the section-B of this chapter.

Secondary, the equation which relates the PWM on-off timing to the Walsh coefficients \(B_i\) is derived. This will be given in the section-C.

Here at this point, if \(B'_i = B_i\) for \(i = 1,3,5,\ldots,\infty\), the PWM wave should coincide with the commanded wave. This is true in theoretical sense. But this kind of coefficients matching can not be achieved actually, because the switching frequency of the PWM inverter can not be infinity. Therefore, we limit the maximum value of \(i\) for the coefficient matching taking account of the switching speed of the power device.

At the last step, the switching timing \(\alpha_i\) of the PWM pattern can be determined directly from the Walsh series coefficients \(B_i\) of the commanded waveform, which is based on a equation derived in the section-D of this chapter.
B. Walsh Series Expansion of Commanded Waveform

In order to make the synthesis simple, it is assumed that the commanded waveform of the PWM inverter output has the property of quarter-wave symmetry. As an example, the case when the output command equals to \( M \sin(\frac{\pi}{2}) \), where \( M \) is modulation ratio, will be explained in what follows. The Walsh series expansion is given as:

\[
M \sin(\frac{\pi}{2}) = \sum_{i=1}^{\infty} \{MB_{2i-1}\cdot \text{sal}(2i-1,t)\}, \quad \text{(9)}
\]

\[
B_{2i-1} = 4 \int_{0}^{\frac{1}{4}} f(t) \cdot \text{sal}(2i-1,t) \, dt \quad i = 1, 2, 3, 4, \ldots \quad \text{(10)}
\]

Since the output command is assumed to be the sine-wave, in the right hand side of (9), the coefficient \( B_{2i-1} \) remains but the coefficients \( A_0 \) and \( A_i \) and \( B_2 \) appeared in (5) and (6) become zero. The calculated values of \( B_{2i-1} \) for \( M=1 \) are shown in Fig. 5.

C. Walsh series expansion of PWM pattern

Fig. 3 shows an example of whole one cycle of Full-bridge PWM inverter waveform which has 3-level. By using the symmetrical property of the PWM pattern, it is enough to calculate only the first quarter cycle. The PWM pattern is setting to be \( f(t) \) which is the function of \( \alpha \).

\[
f(t) = \begin{cases} 1 & 0 \leq t \leq \frac{1}{4} \\ 0 & \text{otherwise} \end{cases}
\]

where \( 0 \leq t \leq 1/4 \).

The Walsh series expansion of \( f(t) \) is given as:

\[
f(t) = \sum_{i=1}^{\infty} \{B_{2i-1}' \cdot \text{sal}(2i-1,t)\}, \quad \text{(12)}
\]

\[
B_{2i-1}' = 4 \int_{0}^{\frac{1}{4}} f(t) \cdot \text{sal}(2i-1,t) \, dt \quad i = 1, 2, 3, 4, \ldots \quad \text{(13)}
\]

D. Proposed Synthesis Method of PWM pattern

In order to derive the synthesis method of the on-off switching function of the PWM inverter, the first quarter cycle of the output command is divided into \( N \) subintervals, where \( N \) should be chosen to be an integer of power of two. The sample quarter cycle of the PWM switching function \( f(t) \) for \( N=4 \) is shown in Fig.6.

To make the switching function \( f(t) \) close to the output command \( M \sin(\frac{\pi}{2}) \), the \( B_{2i-1}' \) in (12) should be set equal to the \( MB_{2i-1} \) in (9), that is,

\[
MB_{2i-1} = B_{2i-1}'. \quad \text{(14)}
\]

As shown in Fig.6, the value of \( \text{sal} \) function is +1 or -1 for \( N \) subintervals, which makes the calculation simple. Letting the value of the \( \text{sal} \) function for each subinterval be \( k_{ij} \):

\[
\begin{array}{cccc}
 k_{11} & k_{12} & k_{13} & k_{14} \\
k_{21} & k_{22} & k_{23} & k_{24} \\
k_{31} & k_{32} & k_{33} & k_{34} \\
k_{41} & k_{42} & k_{43} & k_{44} \\
\end{array}
\]

\[
\begin{array}{cccc}
sal(1,t) & sal(3,t) & sal(5,t) & sal(7,t) \\
\end{array}
\]

\[
f(t) = \begin{cases} 1 & \alpha_1 \leq t \leq \alpha_2 \\ 0 & \alpha_2 < t < \alpha_3 \end{cases}
\]

\[
\begin{array}{cccc}
\alpha_1 & \alpha_2 & \alpha_3 & \alpha_4 \\
\end{array}
\]

\[
\text{PWM}
\]

Fig. 6  PWM design (N=4)

Fig. 5. Calculated values of coefficients \( B_i \) to \( B_{13} \) (M=1)
Rewriting (19) in matrix form,

\[ K_y = \text{sal}(2i-1, \frac{J-1}{4N}). \]  

(15)

According to the property of sal(), the following relations hold:

\[ K_y = K_y, \]

(16)

\[ \sum_{i=1}^{N} (K_y)^2 = N. \]

(17)

From (13) and (14), the following equation holds:

\[ \frac{M}{4}B_{2j-1} = \int_0^{\frac{J}{4N}} f(t) \text{sal}(i, t) dt \]

\[ = K_1 \int_0^{\frac{J}{4N}} f(t) dt + K_2 \int_\frac{J}{4N}^{\frac{2J}{4N}} f(t) dt + \ldots \]

\[ + K_{N(N-1)} \int_\frac{J}{4N}^{\frac{2J}{4N}} f(t) dt + K_{N^2} \int_\frac{J}{4N}^{\frac{3J}{4N}} f(t) dt. \]

(18)

Substituting (11) into (18), the following equation can be obtained:

\[ \frac{M}{4}B_{2j-1} = K_1(T_1 - \alpha_1) + K_2(\alpha_1 - T_1) + \ldots + K_{N(N-1)}(\alpha_N - T_N) + K_{N^2}(T_N - \alpha_N) \]

\[ = K_1(-\alpha_1) + K_2(\alpha_1 - T_1) + \ldots + K_{N(N-1)}(-\alpha_N) + K_{N^2}(\alpha_N - T_N). \]

(19)

where,

\[ T_{2j-1} = \frac{2i-1}{N} \times \frac{1}{4} i=1,2,3,\ldots,N/2. \]

(20)

Rewriting (19) in matrix form,

\[ \frac{M}{4} \begin{bmatrix} B_1 \\ B_3 \\ \vdots \\ B_{2N-1} \end{bmatrix} = \begin{bmatrix} K_1 & K_2 & \ldots & K_{1N} \\ K_2 & K_3 & \ldots & K_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ K_{N1} & K_{N2} & \ldots & K_{NN} \end{bmatrix} \begin{bmatrix} -\alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_N \end{bmatrix} + \begin{bmatrix} T_1 \\ \ldots \\ \ldots \\ -T_{N-1} \end{bmatrix}. \]

(21)

The simple form of the above equation is given as:

\[ \frac{M}{4}[B] = [K] \times [\alpha] + [K] \times [T]. \]

(22)

By the property of the Walsh function, the [K] matrix is a complete orthogonal matrix, and the relations (16) and (17) hold. Taking these properties into account, the inverse matrix of [K] can be simply calculated as:

\[ [K]^{-1} = \frac{1}{N}[K]. \]

(23)

Solving (22) with respect to the [\alpha] and using (23), the switching timing vector [\alpha] can be calculated as:

\[ [\alpha] = \frac{1}{4N}[K] \times [B] \times M - [T]. \]

(24)

The proposed synthesis method is summarized as follows: step-1. calculate \( B_{2j-1} \) matrix by (10), or use data listed in Fig. 5 if \( N \) less than 16. step-2. calculate \( T_{2j-1} \) by (20), step-3. calculate \([K]\) by (15), step-4. then, determine the \([\alpha]\) by (24).

IV. SIMULATION RESULTS

The harmonics of the proposed method and the conventional triangular wave modulation method will be compared in the condition when the switching frequencies of both methods are kept the same.

A. The Case for Pure Sine-Wave Output

In the first quarter of one cycle the number of switching angle of both methods is set to \( N \). For \( N=4 \) the calculation of the Walsh’s calculation method and the conventional triangular wave modulation method are shown in Fig.7 and Fig.8 respectively. For \( N=16 \) the calculation of the Walsh’s calculation method and the conventional triangular wave modulation method are shown in Fig.9 and Fig.10.

As shown in Fig.7(c) and Fig.9(c), the relation between modulation ratio and switching angle of the Walsh’s calculation method are linear. Comparing Fig.7(b) and Fig.9(b), the calculation with larger \( N \) results in less harmonics shifted to higher frequency range. Comparing Fig.7(b) and Fig.8(b), the Walsh’s calculation method can decrease the harmonics (\( 3^{rd}, 5^{th}, 7^{th} \)) more than the conventional triangular wave modulation method. For the calculation by \( N=16 \), the harmonics of both method are almost the same as shown in Fig.9(b) and Fig.10(b).

B. The case for the third harmonic including

In this case, the fundamental wave including the third harmonic is a command waveform for a sample simulation. The amplitude of the third harmonic is setting to 1/6 on the fundamental component, then the command waveform is \( \sin(2\pi \cdot 1)+1/6\sin(6\pi \cdot 1) \). Both calculation methods are compared in the same condition when \( N=16 \). The Walsh’s calculation method is shown in Fig. 11 and the conventional triangular wave modulation method in Fig. 12. The harmonics of both calculations method are shown in Table I.
Fig. 7 Walsh’s calculation waveform when $N=4$

Fig. 8 Triangular’s modulation waveform when switching angle=4

Fig. 9 Walsh’s calculation waveform when $N=16$

Fig. 10 Triangular’s modulation waveform when switching angle=16
From Table I, by the calculation of the triangular wave modulation method we can verify that the amplitude of fundamental and third harmonic matches to the command waveform. The amplitude of harmonics in switching length are symmetry, when the 31st and 33rd harmonics are suppose to be a center of switching.

By the calculation of the propose method, we found the amplitude of fundamental and third harmonic does not match to the command waveform. That is because we limit the number N of Walsh series coefficients (B) to designed the PWM pattern. For matching the amplitude of fundamental and/or the third harmonic, we should adjust infinitely large number of Walsh series coefficients. Also, the amplitude of harmonics in switching length are not symmetry as the amplitude of harmonics calculated by the triangular wave modulation method. The amplitude of harmonics in lower length of the center (25th to 31st) are smaller but the amplitude of harmonics in the higher length of the center (33rd to 43rd) are larger than the amplitude of harmonics calculated by triangular wave modulation method.

### Table I

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V. CONCLUSION

This paper proposes the algorithm to design full-bridge PWM pattern with low harmonics by using the Walsh function. The proposed algorithm process becomes straightforward, in opposition to the algorithm process of the conventional method using Fourier series, which requires an iteration process for searching the PWM pattern. The Walsh function, which has only two level signals of +1 and -1, is suitable to design an algorithm process simpler and faster than that using trigonometric function.

REFERENCES
