Walsh Function Based Reconstruction Method of
Command Waveform in PWM Inverter

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Abstract

This paper proposes a Walsh function based reconstruction-method of command voltage waveform of a PWM inverter. The Fourier harmonics of PWM inverter output voltage show that the PWM switching harmonics spread toward lower frequency range and it makes difficult to reconstruct the PWM command voltage waveform especially in the case when the command voltage waveform has sharp edge. On the other hand, the Walsh harmonics show very good separation between original command voltage component and PWM switching component. Therefore, the Walsh function based reconstruction-method can reconstruct the PWM command voltage waveform correctly. These remarkable features of the proposed method are clarified by several simulation results.

Key Words: Walsh function, Trigonometric function, triangular-wave PWM method

1 Introduction

A modern DSP-based PWM inverter generates the PWM ON-OFF switching pattern using a digital timer. The DSP outputs the PWM ON-OFF pattern but it does not output the PWM command waveform. In this case, in order to find the malfunction of a PWM inverter, it is necessary to reconstruct the PWM command waveform from the PWM ON-OFF pattern. However, the switching harmonics in the PWM ON-OFF pattern make it difficult to reconstruct the PWM command waveform by using Fourier analysis.

Fig.1 The proposed reconstruction-method

Fig.1 shows the proposed reconstruction-method of command voltage waveform of a PWM inverter. The proposed reconstruction-method operates as a Demodulator, which reconstructs the command waveform (Comrec) close to the original command waveform (Com). In order to design the reconstruction-method, it is necessary to thoroughly investigate the PWM operation. This paper shows the analysis of the PWM operation and proposes a demodulation-method of PWM ON-OFF pattern by using Walsh function.

2 Walsh Function

Walsh function consists of rectangular waveforms taking only two values, +1, -1, over one period [0,T] as illustrated in fig.2. Walsh function, wal(n, t/T), has Walsh’s original order, where n is the sequency which is the number of zero-axis crossing for one period. Similar to Trigonometric function, Walsh function can be classified into even-function and odd-function as shows in Table.1. Walsh function can express an arbitrary time function f(t) in terms of Walsh series as shows in (1) and (2).

\[ f(t) = \sum_{n=0}^{\infty} C_n \text{wal}(n, t/T) \quad (1) \]
\[ C_n = \int_0^T f(t) \text{wal}(n, t/T) dt \quad (2) \]
The multiplication theorem of Walsh function is given in (3).

\[
\text{wal}(n_c, \frac{t}{T}) \text{wal}(n_o, \frac{t}{T}) = \text{wal}(n_m, \frac{t}{T})
\]

where

\[
n_m = n_c \oplus n_o
\]

\[\oplus = \text{modulo - 2 addition}\]

(3)

In case when \(n_o < n_c\) and \(n_c = 2^k\), the new generated sequency \(n_m\) is shifted to higher sequency range as shows in fig.3.

3 Investigation of PWM Operation

3.1 Model of Comparator

The triangular-wave PWM method is shown in fig.4 (where \(\text{tri}\) is the triangular-wave). The PWM method generates PWM ON-OFF pattern by using comparator, which makes difficult to investigate the PWM operation because comparator is nonlinear component. The proposed method uses a square-wave instead of comparator as shows in fig.5, which makes it possible to investigate the PWM operation by expanding \(\text{square}(x)\) with the linear function.

By using Fourier series and Taylor expansion, the \(\text{square}(x)\) may also be written as (4) and (5). Therefore, it is possible to make the model of comparator by the multiplication of signal \(x\) as shown in fig.6.

\[
y = \text{square}(x) = \sum_{n=0}^{\infty} (-1)^n \frac{2}{\pi(2n+1)(2n+1)!} x^{2n+1}
\]

\[
y = \frac{2}{\pi} x - \frac{1}{9 \pi} x^3 + \frac{1}{300 \pi} x^5 - \frac{1}{17640 \pi} x^7 + \ldots
\]

where

\[
1 = a_1 + a_3 + a_5 + a_7 + \ldots
\]

(4)

3.2 Operation in PWM Method

Since \(\text{Com}\) is an input signal of PWM method as shows in fig.4, the harmonics of signal \(x = \text{Com-\tri}\) should be the addition of \(\text{Com}\) harmonics and -\tri harmonics. Fig.7 shows the signal \(x^{2n+1}\) and their Fourier/Walsh harmonics in the model of comparator fig.6 (when \(\text{Com}\) is the sin-wave).

<table>
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<tr>
<th>Table.1 Even/Odd function of Trigonometric and Walsh function</th>
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<td>Even Function</td>
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<td>Odd Function</td>
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Fig.2 Walsh function

Fig.3 Walsh multiplication-method

Fig.4 Triangular-wave PWM method

Fig.5 Relation between signal \(x\) and \(y\)

Fig.6 Model of comparator
When looking at Fourier harmonics of signal $x$ in fig.7, its have many harmonics of $tri$ that spread toward low harmonics range (when the low harmonics range is 0 to switching) and its added to the $Com$ harmonics. Therefore, it can not separate the $Com$ harmonics out from the harmonics of PWM ON-OFF pattern by using Fourier analysis.

On the other hand, Walsh harmonics of $Com$ that exist in low harmonics range did not have $tri$ harmonics because $tri$ harmonics are only at switching harmonic and closed to 200th harmonic. Walsh harmonics of signal $x^3$ and $x^5$ in fig.7 show that, $tri$ harmonics did not spread toward low harmonics range. It because of, signal $x^3$ and $x^5$ are the multiplication of signal $x$ that makes harmonics shifted to the higher harmonics range as showed in fig.3. In addition, (4) and (5) told that, the summation of the low harmonics of signal $x^{2n+1}$ are the harmonics of PWM ON-OFF pattern and its equal to low harmonics of signal $x$, which are the $Com$ harmonics.

4 Proposed Reconstruction Method

The above results lead to the observation of PWM operation that, the low harmonics range of PWM ON-OFF pattern does not include $tri$ harmonics and equals to $Com$ harmonics as illustrated in fig.8. In the case when $Com$ is the quarter-wave symmetrical waveform as sin-wave, it’s harmonics are only odd-harmonics of odd-function. Therefore, it can use this property to decrease number of harmonics in (1) and (2) in order to simplify the expression of the command waveform $Com(t)$ as showed in (6) and (7).

\[
Com(t) = \sum_{i=1}^{n} C_{4i-3} \text{wal}(4i-3, \frac{t}{T}) \\
C_{4i-3} = \int_{0}^{T} f(t) \text{wal}(4i-3, \frac{t}{T}) dt
\]
The proposed reconstruction-method of the PWM command waveform is summarized as below.
1) Calculate Walsh harmonics of the PWM ON-OFF pattern \( C_{4,-3} \) by (7).
2) Set \( C_{4,-3} = 0 \), where \( C_{4,-3} \geq \text{switching sequency} \).
3) The PWM command waveform is the reconstruction of \( C_{4,-3} \) by (6).

5 Reconstruction Results

5.1 Type of Carrier and Even Function

Fig.9 shows the variation of PWM ON-OFF pattern in case when the duty-ratio of triangular-wave (Duty) has been changed. From fig.9, when Duty=50[\%], the PWM ON-OFF pattern is the quarter-wave symmetrical waveform, but when Duty=25[\%] or 75[\%], the phase is shifted lead to the PWM ON-OFF pattern is not the quarter-wave symmetrical waveform and it consists of the harmonics of odd-function and even-function. Therefore, when using carrier type2 or type3 as showed in fig.10, the harmonics of even-function of the PWM ON-OFF pattern have been generated and its reproduce the noises including in the reconstruction waveform.

However, when the command waveform (Com) is the quarter-wave symmetrical waveform, harmonics of Com are only the odd-harmonics of odd-function. Therefore, it can separate Com harmonics out from the harmonics of PWM ON-OFF pattern by using (7) even if the harmonics of even-function have been generated.

5.2 Type of Carrier and Harmonics

Fig.11 shows the analysis results when type of carrier and Com have been changed. Types of carrier are showed in fig.10. The Com in fig.11(a-1) is sinusoidal amplitude 0.8, fig.11(b-1) is peak of sinusoidal flattened waveform and fig.11(c-1) is sinusoidal compensated waveform from square-wave amplitude 0.636. The sequency of carrier is setting to be 64. In order to see only the harmonics of Com, the harmonics in fig.11 is setting to be the odd-harmonics of odd-function.

From fig.11, the switching sequency of type1,3 PWM are 64\(^{th}\) and type2 PWM is 128\(^{th}\) that seems to be 2 times of type1,3 PWM. In fact, type2 PWM has the 64\(^{th}\) harmonic as type1,3 PWM but this harmonic is the harmonic of even-function that does not show in fig.11. Fig.12 shows the reconstruction waveforms in case when the type of carrier has been changed. It used lower 64\(^{th}\) harmonics for the reconstruction and the original command waveform showed in fig.11(a-1).

When looking at Fourier harmonics in fig.11, type1 PWM has many switching noises as showed in fig.11(a-4) and its reduce noises including in the reconstruction waveform as showed in fig.12(a). Type2 PWM has the noise harmonics as showed in fig.11(a-6) that higher than the reconstruction harmonics lead to the reconstruction waveform does not have the noises including as showed in fig.12(c). Type3 PWM has the noise harmonics that collected at switching harmonic (64\(^{th}\)) as showed in fig.11(a-8) and its reproduce the noises on the reconstruction waveform fig.12(e).
On the other hand, Walsh harmonics did not have switching component (it has too few of switching component when compared with Fourier harmonics) lead to the reconstruction waveform in fig.12(b),(d),(f) did not have switching noises including and its almost the same waveform.

From the results above, even if the command waveform is the sinusoidal, the switching noises can spread toward low harmonics range of Fourier harmonics when the type of carrier has been changed. On the other hand, the switching noises did not spread toward low harmonics range of Walsh harmonics when the type of carrier has been changed. Therefore, Walsh harmonics can separate the Com component out from the PWM ON-OFF pattern correctly.

5.3 Command Waveform and Harmonics

Fig.13 shows the reconstruction waveform in case when Com has been changed. The Com of fig.13(a),(b) is in fig.11(b-1) and the Com of fig.13(c),(d) is in fig.11(c-1). It used type2 carrier for the modulation and reconstructed waveform by using lower 64th harmonics.

Fourier reconstruction waveform fig.13(a) did not have noises including because noise harmonics in fig.11(b-6) did not spread toward low harmonics range. However, at the half period of waveform in fig.13(c) had noises including because noise harmonics spread toward low harmonics range as showed in fig.11(c-6).

From the observation of fig.11 and fig.13 hold that, switching noises in fig.11(b-6) did not spread toward low harmonics range because the Com (fig.11(b-1)) had just a few of harmonics as showed in fig.11(b-2). However, switching noises in fig.11(c-6) spread toward low harmonics range because the Com (fig.11(c-1)) had much of harmonics as showed in fig.11(c-2).

On the other hand, the switching noises did not spread toward low harmonics range of Walsh harmonics when the Com has been changed or the

<table>
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<th>Fig.11 Fourier/Walsh harmonics of PWM waveform</th>
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| (a) Sinusoidal Waveform  |
| (b) Peak of sinusoidal flattened waveform  |
| (c) Sinusoidal compensated waveform from square-wave  |

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Com has much of harmonics. Fig.13(b),(d) showed that, the Walsh reconstruction waveforms did not have switching noises including and its close to the original command waveform.

6 Conclusions

Walsh harmonics of PWM ON-OFF pattern told that, the PWM command component exists in the left-hand side of switching sequency and switching noise component exists in the right-hand side as showed in fig.8. From section 5.2 and 5.3 hold that, the variation of carrier type and the command waveform did not have effects on the low harmonics range of the Walsh harmonics. Therefore, the proposed Walsh function based reconstruction-method can reconstruct the command waveform close to the original command waveform as showed in fig.12 and fig.13.

Reference